Shutter Drive Application of Ferguson's Paradox

Hycon of Monrovia, California, is one of the leading manufacturers of Aerial Reconnaissance Systems in the United States. In 1962 Hycon, in competition with other leading manufacturers, was awarded a major contract for the manufacture of a framing camera to be known as the KS-72. It was evident that framing cameras would be procured by the United States Government in relatively large numbers and that the other manufacturers were proceeding independently with their own designs to enable them to compete at a future date. In preparation for these coming contests Hycon's management did some advanced planning and organizing. This involved, among other things, the hiring of many new engineers. One of these was Aaron Baumgarten, a native of the East Coast, who was assigned to Hycon's Advanced Product Design Group. Much to his pleasant surprise Aaron discovered upon his arrival in California that Hycon's Sales Manager, Charles Vickery, was a man with whom he had worked closely and successfully on a large proposal some years in the past. Vickery had reported many complimentary things about Aaron's ability as a mechanical designer and these statements had helped to influence the decision to hire him. In discussing various technical problems associated with the design of the KS-72, Vickery told Aaron that Hycon's management was looking for design improvements to enhance the company's position in the up-coming bidding.

He mentioned the Focal Plane Shutter Drive mechanism as a specific area which might be investigated. He explained that the KS-72 was to be equipped with an automatic exposure meter which could vary the exposure while the shutter slit was in transit. Such exposure adjustments are often required in aerial photography; for example, one portion of the film might be exposed by the shutter slit at a time when the lens was looking at a brightly sunlit lake or snow field. A moment later (during the same shutter movement) the shutter slit might be exposing the film to a dark stand of dense forest. Exposure compensation could be made by driving the shutter blades which formed the slit at constant common speed in one direction while simultaneously opening or closing the slit opening in response to a signal generated by a photo-electric cell which was scanning the terrain in advance of the image forming lens. The problem already had one solution, devised by the engineers who had done the original camera design. It was quite an ingenious solution and the engineers were justifiably proud of it. It utilized a Dual Differential as shown in Exhibit 1.

This exhibit shows, schematically, the entire shutter-drive mechanism. The **Shutter Motor** provides the common drive to the curtain slits through one of two clutches depending on the desired direction of drive. The drive train includes the **Dual Differential**. Exhibit 2 shows this portion in somewhat more detail (some of the details originally shown on this drawing are considered proprietary by Hycon and have therefore been deleted). The differential action required to superimpose an opening or closing of the shutter slits on the unidirectional drive of the Shutter Motor is provided by the **Slit Motor**.

This second motor transmits power to the Dual Differential via two gear trains. The gear train driving the differential on the left hand side in Exhibit 1 has an idler gear which does not appear in the drive to the differential on the right hand side. It is by this means that oppositely directed motions are brought to the differentials where they are superimposed on the common drive. Exhibit 3 shows the Dual Differential in schematic form.

Aaron was eager to make a good showing for his new employer and to justify the many good things his friend Vickery had said. His family had not yet moved from the East Coast, so he had the advantage of strong motivation and lots of time to think. For three weeks at the office, and at home well into the night, he tried to devise some combination of linkages, cams, and belt drives which would do the job. However, it all added up to many discouraging hours of effort. Nothing that he thought of was even as good as the original solution—much less "better." Then he remembered a "trick" mechanism which was a favorite with authors of textbooks on kinematics. It was a planetary gear train called Ferguson's Paradox which could produce from one input rotation oppositely directed rotations in two gears mounted on a common center, superimposed on equal rotation in a common direction produced on these same two gears by another input shaft. This was the mechanism he was looking for! It was positive in its action and could (he hoped) be fabricated economically. The prospect of having found a "better" solution after so much effort was very exciting and Aaron happily told his immediate supervisor, Don Shafer, the good news. With tongue in cheek Don said, "Slow down man—you're about to come apart with excitement. It's plain to see that the device you have

sketched out looks better because it is smaller, has fewer parts and is lighter; now try to explain it to me slowly because I don't think it will work."

Aaron explained his idea in detail, using the concept of instant centers to describe the application he had in mind. When he had finished Don said, "You've got me convinced to the point where I'm willing to spend some money on a prototype—but if it doesn't work I'll cut your throat."

Aaron then explained the basic idea to the designer assigned to the job and asked him to design the prototype using only gears which could be found in one of the standard gear catalogs. This was quickly done (see Exhibit 4); the prototype was built (Exhibit 5), and its unique property successfully demonstrated. It appeared to be a unique application and so potentially useful that it was demonstrated to Hycon's Patent Committee. This was a top flight technical group consisting of the Head of Research, Vice President for Engineering, Chief Engineer, Legal Counsel and a Marketing Representative. They in turn requested that a formal disclosure be made and that Aaron prepare a paper deriving the design equations for the mechanism so that others might be able to use the device when it was needed. Aaron did this, preparing Report #989449 which was submitted to the committee for review. This was followed by a patent search which revealed that the application of Ferguson's Paradox to Focal Plane Shutters was indeed unique. A patent application was prepared and filed in the United States Patent Office.

Excerpts from this application, along with some explanatory notes, are shown in Appendix I.
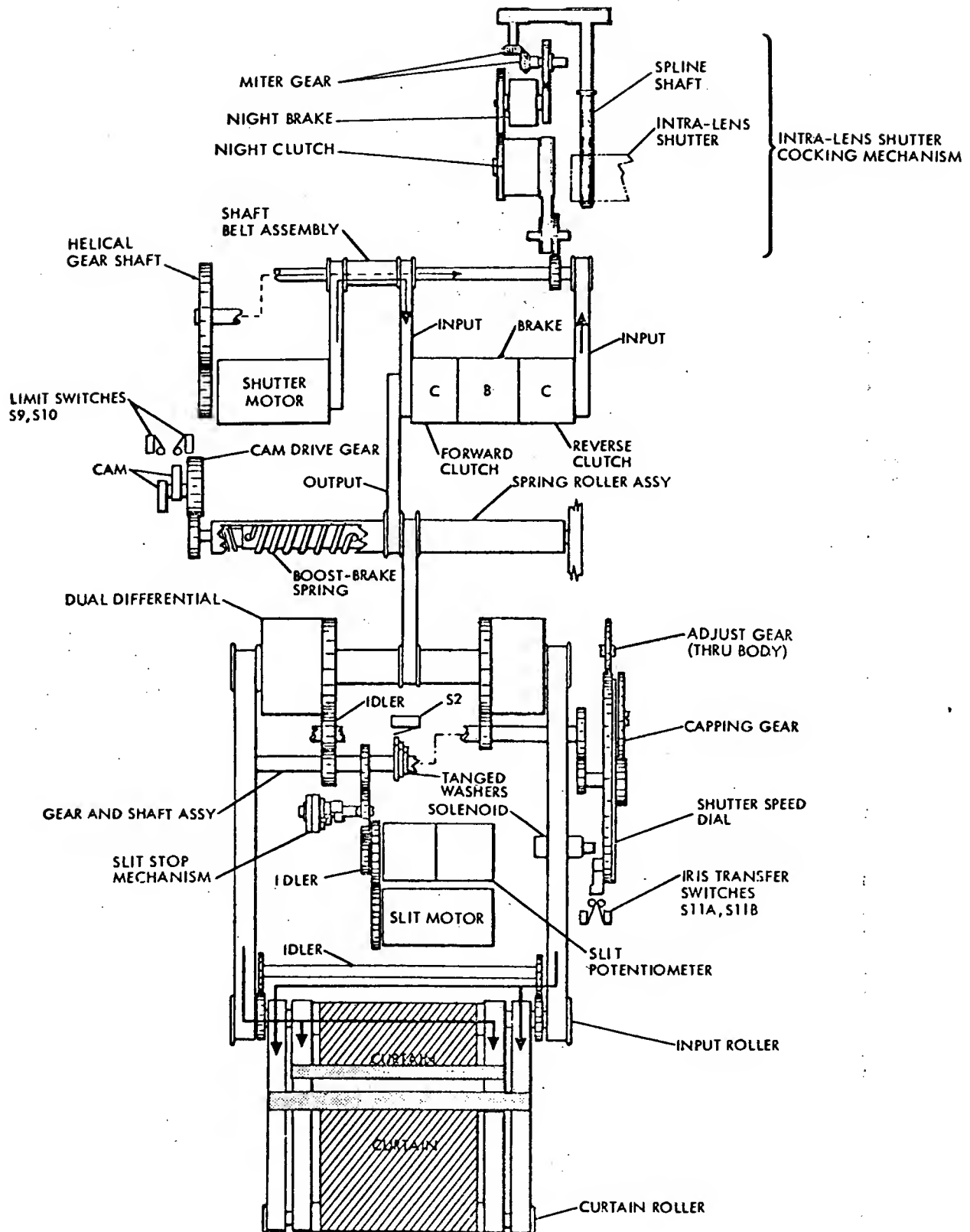
Exhibit 1   Focal Plane Shutter Mechanical Diagram

D

8 REQD ⑦
8 REQD ⑤
2 REQD ㉕
㉘⑯
AR   3 REQD
㉙ TYP BOTH ENDS
㉚

6 REQD ⑳
2 REQD ③

2 REQD ⑭
2 REQD ⑫
2 REQD ⑮
16 REQD ㉑

㉓

⑧ 2 REQD   ⑰ 2 REQD   ②   ⑩
⑬
⑰ REF
⑪   ㉙ ㉚   ⑯ ㉘ REF   ②
⑭   ㉔   B R
A'   ⑯ ㉘ REF
LINE A ⑬   /12\
0001
LINE A   ㉖ 4 REQD
A'   ㉓ REF
④
①
⑲ 2 REQD
⑨ 2 REQD /14\
⑥ 8 REQD

B
A
A
B

⑪ ㉗

VIEW B-B /14\
TYPICAL /13\

-0001 F/NI
0001 F/NI

VIEW A-A
VIEW A'-A'

/12\ MARK FOUR DIGIT SERIAL NUMBER, STARTING WITH 0001, APPROX WHERE SHOWN IN .12 HIGH NORMAL GOTHIC CHARACTERS, USING RED INK PER TT-I-558 AND COVER WITH CLEAR VARNISH PER MIL-V-173.

/11\

/10\ LOCTITE CORP, NEWINGTON, CONN.

/9\

/8\ AS SUPPLIED BY SPIROLOX DIV. OF RAMSEY CORPORATION, ST. LOUIS, MO.

/7\
/6\
/5\
/4\

/3\ AS SUPPLIED BY SPLIT BALL BEARING DIV. OF MINIATURE PRECISION BEARINGS, INC. LEBANON, NEW HAMPSHIRE.

/2\ AS SUPPLIED BY MINIATURE PRECISION BEARINGS INC. KEENE, NEW HAMPSHIRE OR EQUIVALENT.

1. INTERPRET DRAWING IN ACCORDANCE WITH STANDARDS PRESCRIBED BY MIL-D-70327.

NOTES: UNLESS OTHERWISE SPECIFIED

KS-72

NEXT ASSEMBLY | USED ON
APPLICATION

HMC 1558 REV. 6/64

		ZONE	LTR	REVISIONS		
				DESCRIPTION	DATE	APPROVED
				SEE SH 1 FOR CHANGES		

D

Exhibit 2
Dual Differential Assembly,
Focal Plane Shutter KS-72 Camera

-/ QTY. REQD.	FIND NO.	REF DES.	PART OR IDENTIFYING NO.	NOMENCLATURE OR DESCRIPTION	MATERIAL OR NOTE	SPECIFICATION	ZONE NO
A/R	30		GRADE B	SEALING COMPOUND		MIL-S-22473	
A/R	29		GRADE N FORM R	PRIMER		MIL-S-22473	
AR	28			LOCTITE KEY FIT	10		
4	27			PIN, SPRING, .062 X .188 LG	H58667-197		
4	26			SCREW, FIL H, 440 X .188 LG	MS 35275-1 EXCEPT BLACK OXIDE		
2	25			BEARING	3 7		
1	24			RING, RETAINING	8 9		
4	23			PIN, DOWEL, .062 X .25 LG			
8	22			SCREW, FH, 2-56 X .25 LG			
16	21			BEARING	2 6		
6	20			BEARING	2 5		
	19			BEARING	3 4		
	18						
2	17			COVER			
3	16			KEY			
2	15			FLANGE			
2	14			RETAINER			
1	13			PULLEY			
2	12			PULLEY			
1	11			GEAR, LH			
1	10			GEAR, RH			
2	9			SPACER			
2	8			HOUSING			
8	7			SPACER			
8	6			GEAR			
8	5			SHAFT			
1	4			SHAFT			
2	3			GEAR			
1	2			RETAINER			
1	1			SHAFT			

PARTS LIST

UNLESS OTHERWISE SPECIFIED
DIMENSIONS ARE IN INCHES

CONTRACT NO.

TOLERANCES   SURFACES

DRAWN *Andy Brovitt*   DATE 2-9-66

DESIGN   2-23-66

CHECK *E. Smith*   2-22-66

ENGINEER   2-23-66

PROJECT *B. Goern...*   2-23-66

√ MICRO-INCHES

DO ... SCALE DRAWING

(hp) **hycon** mfg company

MONROVIA, CALIFORNIA

TITLE:
DUAL DIFFERENTIAL ASSEMBLY
FOCAL PLANE SHUTTER ASSY
KS-72 CAMERA

A

SIZE	CODE IDENT NO.	DWG NO.
C		

SCALE FULL | UNIT WEIGHT ——— | SHEET 11

B

INPUT DIFFERENTIAL GEAR

INPUT SUN GEAR

PLANETARY PINION -
SHOWN ROTATED OUT OF POSITION

OUTPUT PULLEY

Exhibit 3a
Spur Gear Differential
(two of these make up a dual differential)

PLANETARY PINIONS

INPUT DIFFERENTIAL GEAR

INPUT SUN GEAR

OUTPUT GEAR
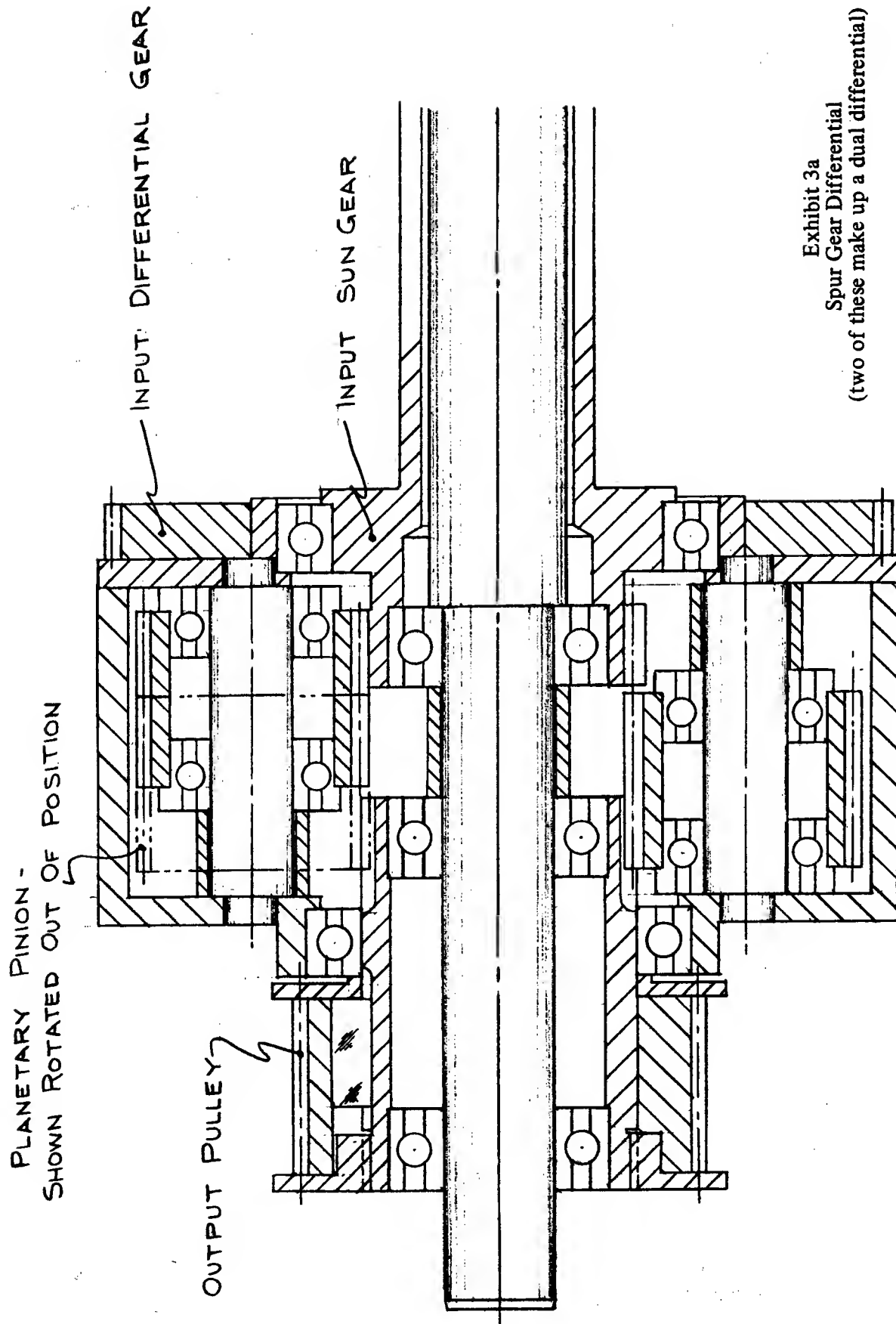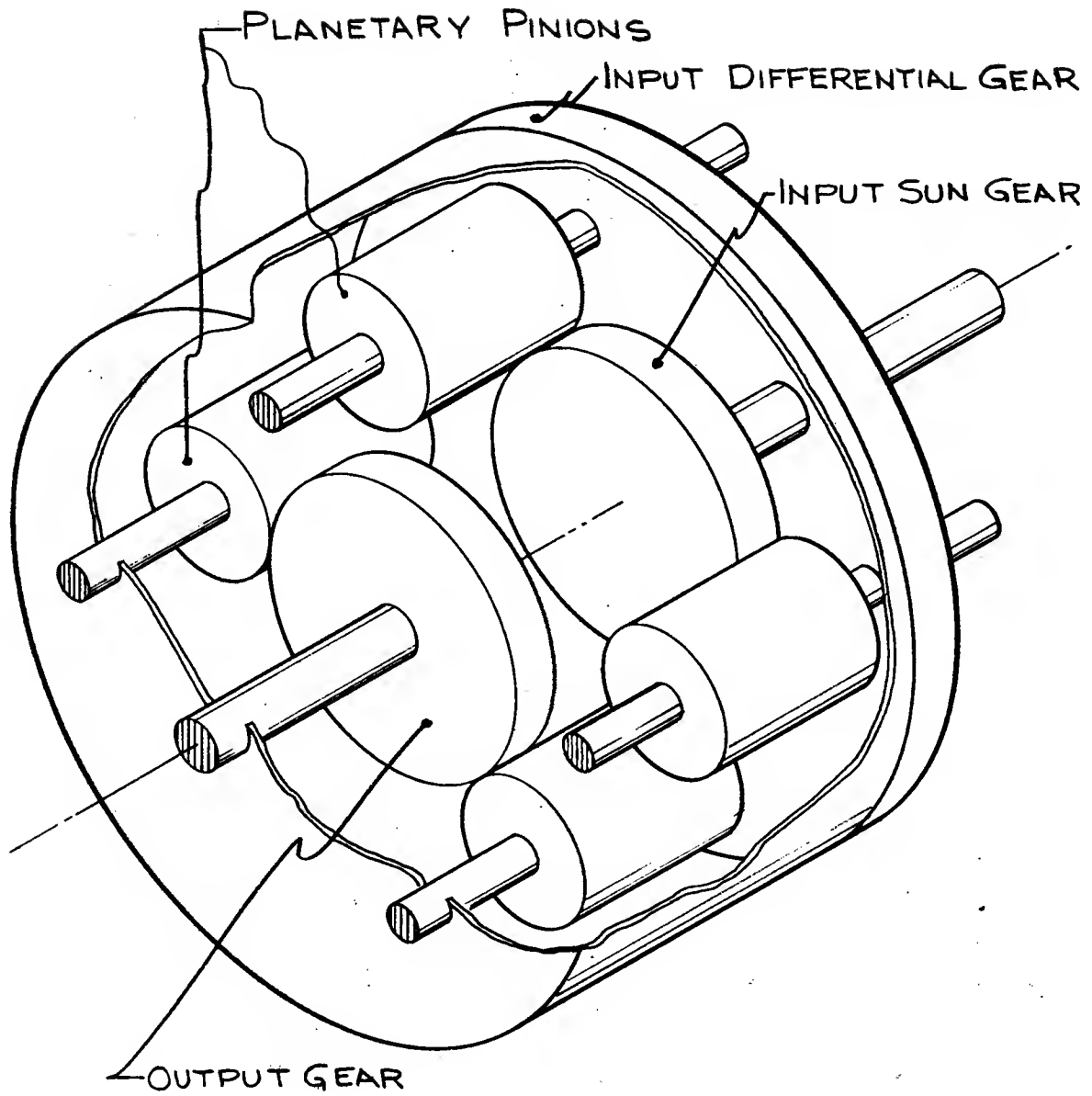
Exhibit 3b
Schematic of a
Spur Gear Differential

V46A-3
INPUT FOR SYNCHRONOUS OPPOSITE OUTPUT

AL-13

AL-16
INPUT FOR SYNCHRONOUS UNIDIRECTIONAL OUTPUT

1.500

.500

CF187BA-0250

D48A-64
(64T-64P)
CF187BA-0250

CF187BA-0250

V48A-3

CF126B-0148
CF126B-0148

CF187A-0125

E46A-36

F46A-36 (36T-80P)
D46A-36
(36T-64P)

.500

F46A-36 (36T-80P)
CF187BA-0250

OUTPUT PULLEYS

CF187BA-0250

(36T-72P) E44A-36
(36T-80P) F46A-36
CF187A-0125

D46A-36

4.00

.500

CF187BA-0250

E46A-36
(36T-72P)

E46A-36
(36T-72P)

D48A-128
(128T-64P)

BUSHINGS-
ATLAS BRASS FOUNDRY
1901 SANTA FE AVE.
LOS ANGELES, CALIF.

GEARS & MISC.
PERFECT GEAR & INSTRUMENT CO.
339 SO. ISIS AVE.
INGLEWOOD, CALIF.

SHAFTS TO BE STAINLESS STEEL

F46A-44
(44T-80P)
CF187BA-0250

D46A-28
(28T-64P)

D46A-28
(28T-64P)
CF187A-0125

CF187A-0125

F46A-44
(44T-80P)
CF187-0322

CF187A-0125

Exhibit 4
Assembly Drawing of Prototype

Exhibit 5
Prototype,
Shutter Drive Application
of Ferguson's Paradox

APPENDIX I

Note on Patent Application*

As is usual in patent applications the Patent drawing was prepared from the Prototype. The description is very detailed and specific, as required by the Patent Laws, and the claims begin with a very broad one. The next claim is very similar but is intended to be narrower in scope. Succeeding claims further narrow the scope of the claimed invention, as can be seen from Claim 8.

A patent application must contain a complete description of the invention. The description usually includes a series of drawings and descriptive material called the specification which teaches the manufacture and use of the invention. The applicant must also submit a claim or claims defining the monopoly which he hopes to obtain. In submitting claims routine practice is to submit multiple claims covering the invention which differ in the breadth of the claimed monopoly. This practice has two purposes:

First, if the search by the Patent Office reveals prior art which would invalidate a proposed claim, narrower claims are before the Examiner for possible allowance.

Second, the patentee desires claims of varying breadth in his patent since a court, in litigation, may invalidate broader claims, leaving narrower claims in full force and effect.

In submitting claims a patent attorney will present the broadest claims which he feels make a distinction over prior art which he is aware of, giving the benefit of any doubt to his client and relying on the adversary system and on the Patent Office Examiner to exclude or disallow improper claims.

Excerpted below are the abstract of the invention, a portion of the detailed description of the device, the first, second and eighth Claims, and the figures which accompanied the application.

*This note was prepared by the Hycon patent attorney.

Appendix I

Excerpts from Aaron Baumgarten's Patent Application

Abstract of the Invention: Focal plane shutter apparatus having a shutter slit which is adjustable from a remote location. A pair of shutter curtains is utilized to form the shutter slit, and a transmission mechanism is controlled to drive the curtains simultaneously at the same speed and alternatively at different relative speeds.

TO WHOM IT MAY CONCERN: Be it known that I, AARON BAUMGARTEN, a citizen of the United States of America and residing in the County of San Mateo, State of California, have invented a new and useful SHUTTER DEVICE, of which the following is a specification:

The present invention relates to cameras, and more particularly to means for driving the shutter which controls the exposure of the film.

There are a wide variety of photographic cameras for exposing a light sensitive film to form a latent image thereon. The duration of the exposure is controlled by a shutter which is normally maintained closed, thereby preventing light from reaching the film. When an exposure is made the shutter is momentarily opened for a predetermined time interval. The amount of the exposure is controlled by the duration of the interval that the shutter is open.

One common type of shutter frequently employed for this purpose is the so-called "focal plane" shutter. Such a shutter employs one or more opaque curtains which are disposed substantially in the focal plane and immediately adjacent the film. The ends of the two curtains are closely spaced but separated from each other by a small distance whereby an open slit is formed. When the two curtains are drawn across the front of the film this slit is also moved across the film. This allows a narrow band of the image-forming light to travel across the film and to expose the film.

If the speed at which the curtains travel is constant and the slit width is constant, then the exposure is constant. The duration of the exposure is a function of the width of the slit and the velocity at which the curtain travels. Accordingly, by moving one of the curtains relative to the other curtain, the width of the slit may be varied, controlling the duration of the exposure even though the curtains are always driven at a constant speed.

Many different types of drive means have been proposed in the prior art for driving the curtains across the face of the film and for controlling the size of the slit. However, in spite of a considerable amount of time and effort being devoted to the development of satisfactory focal plane shutters, it is still extremely difficult, if not impossible, to make a simple focal plane shutter capable of providing an accurate exposure.

The simplest and most accurate drive mechanisms simultaneously move both of the curtains at a constant speed. Since the two curtains are operatively coupled together it has been necessary to provide some form of release in the drive mechanism to permit one or both of the curtains to be moved when the width of

the slit is to be adjusted. This, however, tends to complicate the drive mechanism. Moreover, once the drive mechanism has been actuated and the curtains are in motion it has been very difficult to modify the size of the slit, without interrupting the motion, to vary the exposure in all or particular portions of the image. It may thus be seen that prior art types of focal plane shutters have not been entirely satisfactory.

The present invention provides means for overcoming the foregoing disadvantages and limitations. More particularly the present invention provides a focal plane shutter and drive mechanism therefor, which are simple and reliable and yet permit the motion of the curtains and the size of the slit therebetween to be varied independently of each other. In addition, the means provided permits varying the width of the slit whereby the magnitude of the exposure may be varied in different regions of the image.

The embodiment of the present invention disclosed herein, this is accomplished by providing a pair of curtains mounted on separate drums or rollers that support the curtains and allows them to be transported across the focal plane. The drive mechanism includes an epicyclic drive train that is coupled to the drums or rollers for driving them simultaneously. The epicyclic drive train also includes a pair of separate inputs that are coupled to a pair of separate drive motors.

If the first drive motor is operated and the second drive motor retains the second input fixed, the two curtains move relative to each other whereby the width of the slit is varied to the desired size. If the second drive motor drives the second input and the first drive motor retains the first input fixed, both curtains are simultaneously transported in the same direction at the same speed whereby the slit will move across the film and a uniform exposure is made.

Either of the two inputs may be driven without affecting the other and it is no longer necessary to provide any sort of a release. If, for any reason it is desired to vary the amount of the exposure in different portions of a picture, the two inputs may be operated simultaneously. The effective speed at which the slit is travelling will remain constant but the width of the slit can be varied to increase or decrease the exposure of certain areas of the image.

These and other features and advantages of the present invention will become readily apparent from the following detailed description of a single embodiment thereof, particularly when taken in connection with the accompanying drawings wherein like reference numerals refer to like parts.

FIG. 1 of this patent Application is a perspective view of a preferred embodiment of adjustable shutter apparatus according to the present invention;

FIG. 2 is a plan view of a drive transmission of the shutter apparatus of FIG. 1; and

FIG. 3 is a side view, partially in cross-section, of the drive transmission shown in FIG. 2.

Turning to FIG. 1, a shutter 10 is adapted to be positioned immediately adjacent to a photographic film surface for traversing the focal plane of a camera. The

shutter 10 includes a first curtain 12 and a second curtain 14, which may be of any desired type of material but are normally made from a relatively thin, flexible material which is completely impervious to light.

In this example of a focal plane shutter, the first curtain 12 is adapted to be rolled around a first idler roller 16, and the second curtain 14 is adapted to be rolled around a second idler roller 18. A free edge of the first curtain 12 is attached to a first bar member 20, while a free edge of the second curtain 14 is attached to a second bar member 22. The ends of the first bar member 20 are attached to a first pair of endless belts 24, 26, while the ends of the second bar member 22 are attached to a second pair of endless belts 28, 30. Each of the endless belts are positioned in the camera so that when they are caused to move, the bar members 20, 22 will traverse the camera focal plane. The bar members 20, 22 are positioned parallel to each other, and the separation between them describes a shutter slit 32.

Both of the drive pulleys 34, 38, therefore, are caused to rotate simultaneously in the clockwise direction. If desired, both of the drive pulleys 34, 38 can be caused to rotate simultaneously in the counterclockwise direction by applying counterclockwise rotation to the sun drive transmission gear 96 by the second motor. In either case, the angular velocities of the first and second drive pulley gear 84, 86 must be identical in order to provide identical speeds to the drive pulleys 34, 38. This is accomplished by appropriate design of the individual spur gears included in the transmission 50, by techniques which are well known to the transmission art.

When it is desired to adjust the width of the shutter slit 32, the second motor acts as a brake to prevent rotation of the sun drive gear 60, and the first motor is caused to rotate the spider gear 56 by rotating the spider drive gear 88. For example, if the spider drive gear 88 is caused to rotate in a clockwise direction as shown in the drawing, the spider gear 56 will rotate in a counterclockwise direction. The first planetary gear 54 rotates epicyclically counterclockwise with respect to the first sun gear 52, which is held stationary. Corresponding rotation is thereupon transmitted to the second planetary gear 78 and to the third planetary gear 80.

The epicyclic counterclockwise rotation of the second planetary gear 78 produces counterclockwise rotation of the second sun gear 66. Since this latter gear is rigidly mounted on the first shaft 58, the second drive transmission gear 74 rotates in a corresponding counterclockwise direction to produce clockwise rotation of the second drive pulley gear 86, causing the attached second drive pulley 38 to rotate in the clockwise direction.

The epicyclic counterclockwise rotation of the third planetary gear 80 produces clockwise rotation of the third sun gear 70, causing the first drive transmission gear 72 to rotate in a clockwise direction. Counterclockwise rotation is thereupon produced in the first pulley drive gear 84, causing the first drive pulley 34 to rotate in counterclockwise direction.

The drive pulleys 34, 38, therefore are caused to rotate simultaneously in opposite directions which, in the case described, causes the width of the shutter slit 32 to decrease. Alternatively, the width of the shutter slit can be caused to increase by applying counterclockwise rotation to the spider drive gear 88 by the first motor.

If desired, a shutter slit 32 of varying

width can be controlled to traverse the camera focal plane. This condition is accomplished by operating both the first and second motors simultaneously, but controlled separately, to ultimately rotate the drive pulleys 34, 38 in the same direction but at varying speeds. In this manner, the amount of exposure for different portions of a photograph can be varied.

Thus there has been shown a preferred embodiment of a focal plane shutter having an adjustable exposure slit, including means for causing the shutter slit to traverse a predetermined plane as well as means for adjusting the width of the shutter slit. Other embodiments of the present invention and modifications of the embodiment herein presented, may be developed without departing from the essential characteristics thereof.

Accordingly, the invention should be limited only by the scope of the claims appended below.

What is claimed as new is:

1.   A shutter device comprising the combination of:

   (a)   a curtain transport,

   (b)   a pair of curtains mounted on said transport, said curtains having a pair of adjacent ends positioned to form an opening therebetween,

   (c)   drive means coupled to said curtain transport for driving the transport and moving said curtains,

   (d)   said drive means including a first portion for simultaneously moving both of said curtains in the same direction and a second portion for moving at least one of said curtains relative to the other.

2.   A shutter device comprising the combination of:

   (a)   a pair of curtains, said curtains having a pair of adjacent ends positioned to form an elongated opening therebetween,

   (b)   a curtain transport coupled to said curtains to be moved,

   (c)   first means coupled to said curtain transport for simultaneously driving both of said curtains in the same direction to move the opening, and,

   (d)   second means coupled to said curtain transport for moving at least one of said curtains relative to the other to change the the size of the opening.

8.   A camera shutter comprising the combination of:

   (a)   a curtain transport,

   (b)   a first curtain mounted on the transport,

   (c)   a second curtain mounted on the transport adjacent to said first curtain, said curtains having a pair of juxtaposed ends positioned adjacent to each other to form an opening therebetween, the size of said opening being determined by the relative positions of said curtains,

   (d)   an epicyclic train,

   (e)   a pair of inputs to said train,

   (f)   a pair of outputs to said train, one of said outputs being coupled to one of said curtains and the other of said outputs being coupled to the other of said curtains,

(g) first drive means coupled to one of said inputs for simultaneously driving both of said curtains in the same direction, and

(h) second drive means coupled to the other of said inputs for driving at least one of said curtains relative to the other of said curtains.
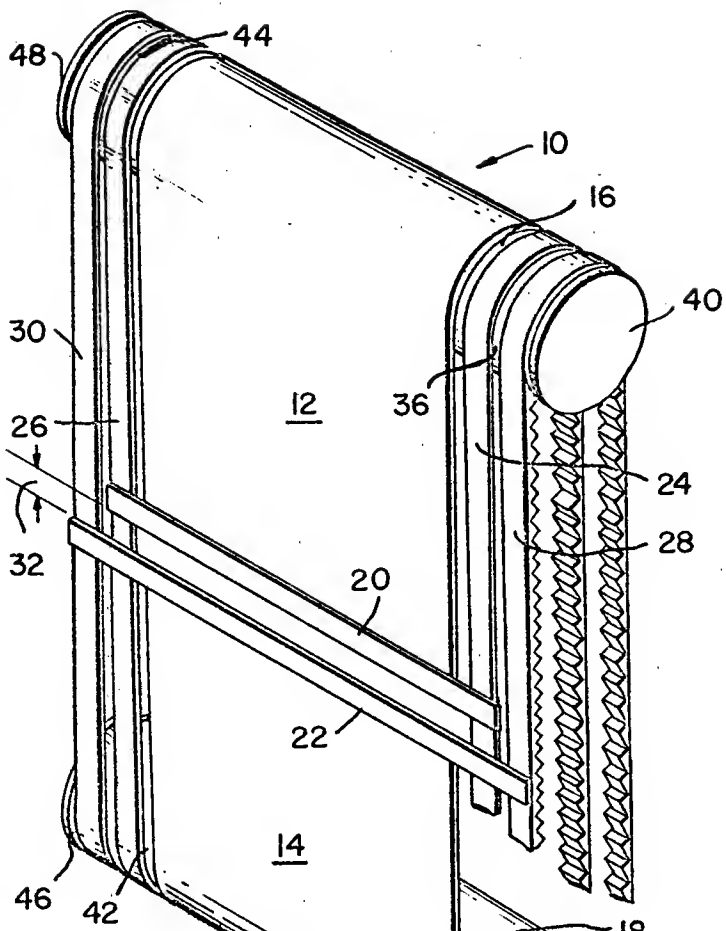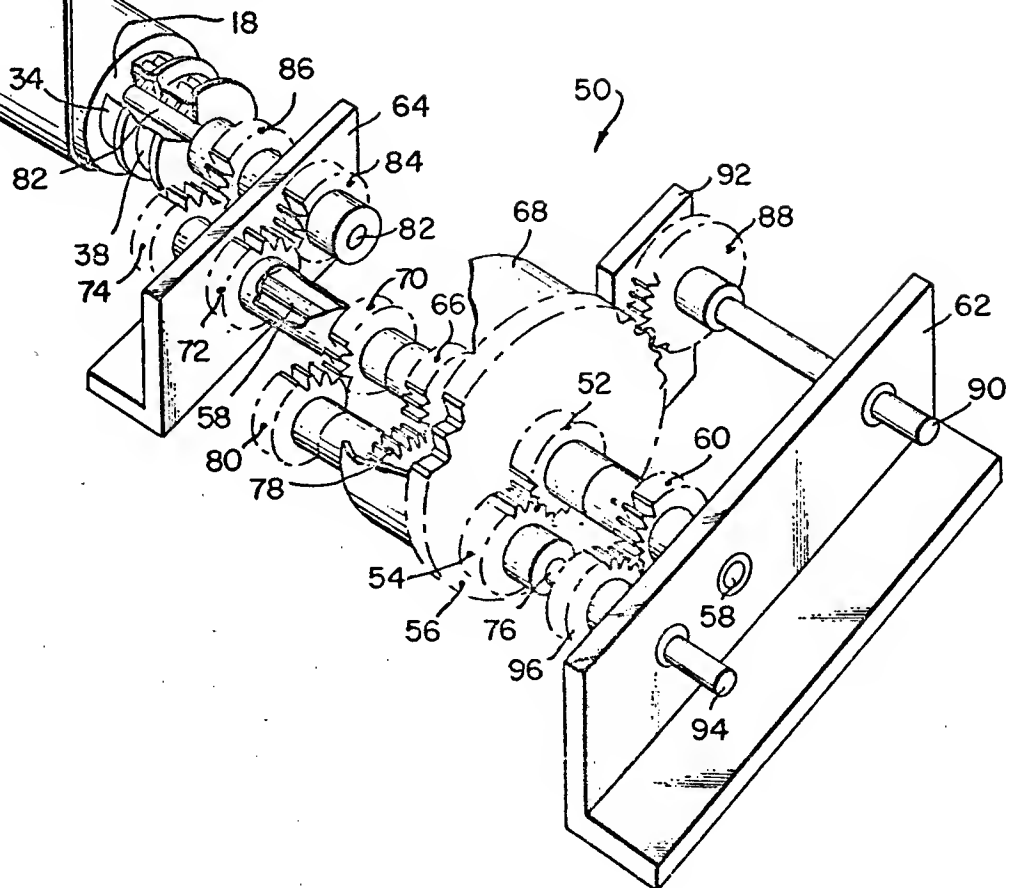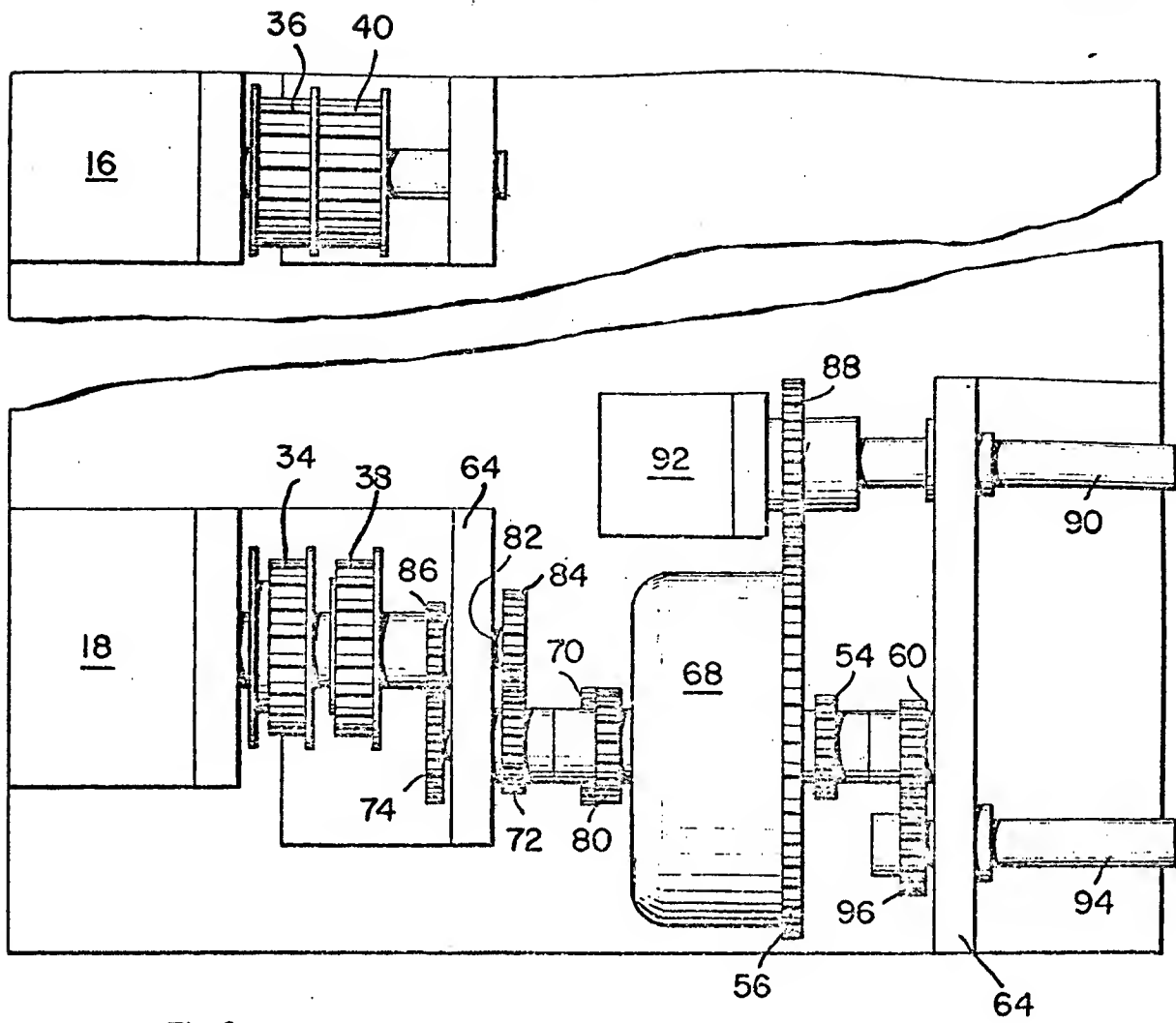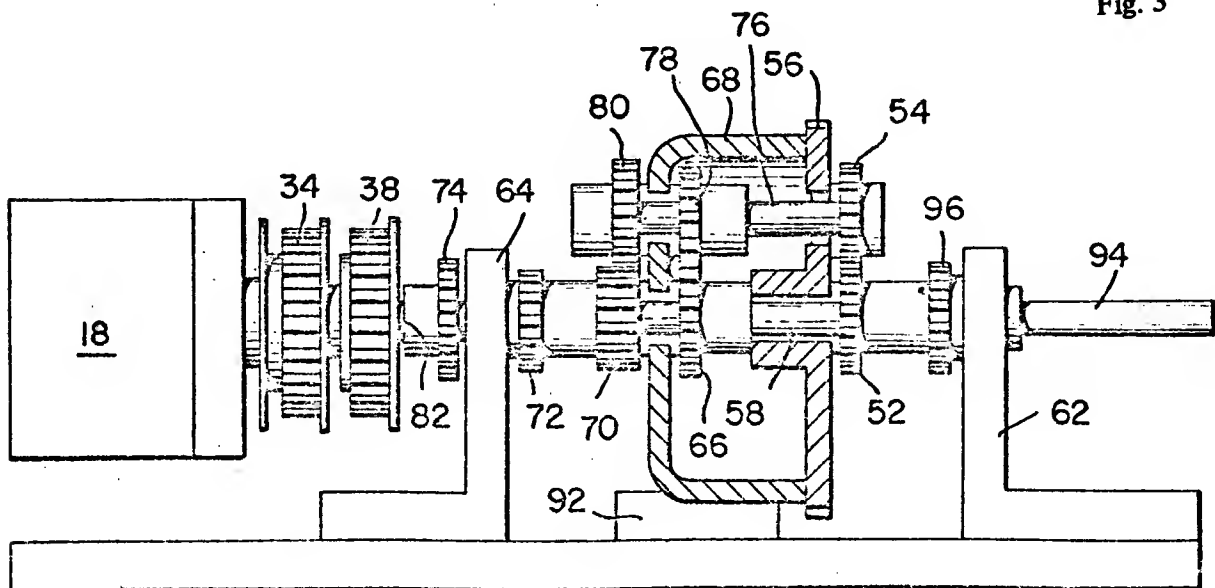
Fig. 1

Fig. 2

Fig. 3

Instructor's Note

This case is intended for graduate and undergraduate courses in kinematics and machine design; it deals with an application of two important kinematic concepts: a) Instant Centers and b) Differential Gear Trains.

I. An explanation of the action of this particular Differential Gear Train in terms of Instant Centers has been included in this instructor's note. The kinematic analysis in terms of the equation for the Train Value of a Differential Gear Train has also been included. (This last is the Hycon report prepared by Aaron Baumgarten.) Both of these analyses could be assigned as student exercises in an undergraduate course in kinematics after both topics, Instant Centers and Differential Gear Trains, have been studied. This explanation draws heavily upon Kenedy's Theorem. However, the task of finding Instant Center 41 (not shown in Exhibit 6) presents an opportunity to demonstrate that the theorem is not uniquely applicable in every circumstance.

II. The author was not able to find another solution he considered satisfactory. However, some talented student (?) may be willing to undertake, as a "project" the task of trying to synthesize some other means of solving the problem.

III. At the graduate level it is suggested that the students be assigned the task of designing a "Ferguson" drive to satisfy some postulated output velocity and power requirement.
A - As a corrollary, the problem may ask for an identification of the most highly stressed gear and for a speed/torque limitation if any exists.

IV. Refer to the schematic layout which shows the original DUAL DIFFERENTIAL solution. Ask the students to replace the DUAL DIFFERENTIAL with FERGUSON'S PARADOX and to evaluate the "improvement" in the over all design.

V. Does the prototype represent a good "production" design or can the number of piece parts be reduced? A "design-for-production" should be considered as a project assignment.

APPLICATION OF "FERGUSON'S PARADOX" TO SLIT

WIDTH CONTROL IN FOCAL PLANE SHUTTERS

"Ferguson"s Paradox" is the name given to a reverted planetary
spur gear system which has possible application to the problem of focal-
plane shutter slit-width control and curtain drive. The actions produced
by the planetary system are positive, and not subject to slippage. The
elimination of friction elements in the transmission improves the reliabil-
ity of setting exposure and maintenance of curtain velocity in a focal-
plane shutter. Also, a thoughtful design effort may be expected to result
in an inexpensive transmission.

In detail, the motions produced by the transmission are as follows:
(a) When the spider (i.e., planetary arm) is driven and the sun gear
is held stationary, two gears mounted co-axially with the sun gear (but
not fastened to it) are caused to rotate in opposite directions. This
action may be utilized in setting the shutter slit-width.
(b) When the spider (i.e., planetary arm) is held and the sun gear is
driven; the two gears (mounted coaxially with the sun gear) will rotate
in the same direction as the sun gear. This action may be used to drive
the shutter during the picture-taking process.

The magnitudes of the relative angular velocities of the sun gear
and the two co-axial gears are determined by the details of design
illustrated below.

DESIGN EQUATIONS

Exhibit 4 is a drawing of a breadboard being built to demonstrate
the principle of "Ferguson's Paradox", and may be used, in a general sense,
for following the design equations utilized below in an illustrative example.

Where:

T.V. = Train Value of transmission when considered to be a non-planetary system. (Attention must be given to the algebraic sign.) The value of T.V. found this way may then be used in the epicyclic equation (1).

C.D. = Center Distance

ω = Angular Velocity

N = Number of teeth (subscripts A,B,C,J,K and L refer to different gears)

P = D.P. = Diametral Pitch (subscripts refer to different values)

I = Integer

(1) $$T.V. = \frac{\omega_{LAST} - \omega_{ARM}}{\omega_{FIRST} - \omega_{ARM}}$$

(2) $$C.D. = \frac{N_J}{2 \times (D.P.)_1} + \frac{N_A}{2 \times (D.P.)_1} = \frac{N_K + N_B}{2 \times (D.P.)_2} = \frac{N_L + N_C}{2 \times (D.P.)_3}$$

$$N_J = N_K = N_L = N_B$$

$$\left. \begin{array}{l} N_A = N_J - I \\ N_C = N_J + I \end{array} \right\} \quad I = 1,2,3,4,\dots$$

Using: $P_1 = (D.P.)_1$; $P_2 = (D.P.)_2$; $P_3 = (D.P.)_3$,

we have:

$$C.D. = \frac{N_J + N_J - I}{2P_1} = \frac{N_J + N_J}{2P_2} = \frac{N_J + N_J + I}{2P_3}$$

Therefore we get:

$$C.D. = \frac{2N_J - I}{2P_1} = \frac{2N_J}{2P_2} = \frac{2N_J + I}{2P_3}$$

or:

$$\frac{P_1}{P_2} = \frac{2N_J - I}{2N_J} \qquad \text{and:} \qquad \frac{P_2}{P_3} = \frac{2N_J}{2N_J + I}$$

The value assigned to N_J is limited by the requirements of involute geometry.

The value assigned to $\underline{I}$ is determined by N_J and the requirements of the particular system under consideration.

The values assigned to P_1, P_2 and P_3 are primarily determined by the strength requirements of the system.

DESIGN EXAMPLE

To illustrate the action of the mechanism, let us try to design a representative gear train which has a center distance (C.D.) = 0.500 and N_J = 18 minimum.

$$0.500 = \frac{2 \times 18 - I}{2P_1} = \frac{36 - I}{2P_1}$$

$P_1 = 36 - I$

If I is 1, then $P_1 = 35$

$P_2 = 2 \times 18 = 36$

$P_3 = 2 \times 18 + 1 = 37$

Note: in the prototype N_J = 36 and I = 8, therefore $P_1 = 64$, $P_2 = 72$, $P_3 = 80$.

and

$N_A = 18 - 1 = 17$

$N_C = 18 + 1 = 19$

Using Equation (1) , we first calculate T.V. for train B-K-J-A as:

$$\frac{18}{18} \times \frac{18}{17} = + \frac{18}{17} \quad .$$

For this train, "B" is the first gear and "A" is the last gear. Holding sun gear "B" stationary, rotate spider one revolution.

$$\frac{18}{17} = \frac{\omega_{LAST} - 1}{0 - 1}$$

$$17\omega_{LAST} - 17 = -18$$

$$\omega_{LAST} = -\frac{1}{17} = A$$

Now, calculate T.V. for train B-K-L-C as $\frac{18}{18} \times \frac{18}{19} = \frac{18}{19}$.
For this train, "B" is the first gear and "C" is the last gear.
Holding sun gear "B" stationary, rotate spider one revolution (same direction as before):

$$\frac{18}{19} = \frac{\omega_{LAST} - 1}{0 - 1}$$

$$19\omega_{LAST} - 19 = -18$$

$$\omega_{LAST} = +\frac{1}{19} = C$$.

This counter-rotation of gears "A" and "C" will occur simultaneously as the spider is rotated in either direction.

Now, hold the spider fixed and rotate sun gear "B" .

For Train B-K-J-A   **For Train B-K-L-C**

$$\frac{18}{17} = \frac{\omega_{LAST} - 0}{1 - 0} \qquad\qquad \frac{18}{19} = \frac{\omega_{LAST} - 0}{1 - 0}$$

$$\omega_{LAST} = \frac{18}{17} = A \qquad\qquad \omega_{LAST} = \frac{18}{19} = C$$.

From this we can form the ratio:

$$\frac{\omega_C}{\omega_A} = \frac{\frac{18}{19}}{\frac{18}{17}} = \frac{17}{19}$$.

Since these two gears drive the shutter curtain during the picture-making process, their angular velocities must be identical.

This is achieved by having gears "A" and "C" each mesh with independent gears, each having 18 teeth (i.e., N_J) . This means that during the picture-taking process, the gear meshing with "A" will turn at:

$$\frac{18}{17} \text{ x } \frac{17}{18}$$

or 1 rev/rev of sun gear "B" , and that the gear meshing with gear "C" will turn at:

$$\frac{18}{19} \text{ x } \frac{19}{18}$$

or 1 rev/rev of sun gear "B" .

During the planetary phase of the action, we would now have, on the side of gear "A" :

$$- \frac{1}{17} \text{ x } \frac{17}{18} = - \frac{1}{18}$$

and on the side of gear "C" :

$$+ \frac{1}{19} \text{ x } \frac{19}{18} = + \frac{1}{18}$$

From this we see that the two curtain segments can be moved apart (or closer together, as the case may be) in synchronism during slit-width adjustment by driving the spider and holding the sun gear stationary. During the picture-taking process, the two curtain segments can be moved in the same direction at the same speed by holding the spider stationary and driving the sun gear.

Action of Ferguson's Paradox
Explained in Terms of Instant Centers

By definition, an Instant Center of Velocity is "The point about which
one body rotates relative to another for any particular phase being considered.
It is a double point. The bodies have no linear velocity relative to each
other at this point. At this point the two bodies have the same linear
velocity relative to any other body."

Instant Centers are of three types: fixed, permanent and of a nature that
is neither fixed nor permanent.

The location of fixed and permanent centers is frequently obvious and
may be found by inspection. A useful tool for finding the last type, the
kind which is neither fixed nor permanent, is a kinematic theorem known as
"Kenedy's Theorem" which states that "If three bodies have plane motion their
instant centers lie on a straight line". Also if we know the location of an
instant center of rotation for some machine element and the velocity of any
point on that machine element, we can find the velocity of any other points
on that element by a simple graphical construction. For example:
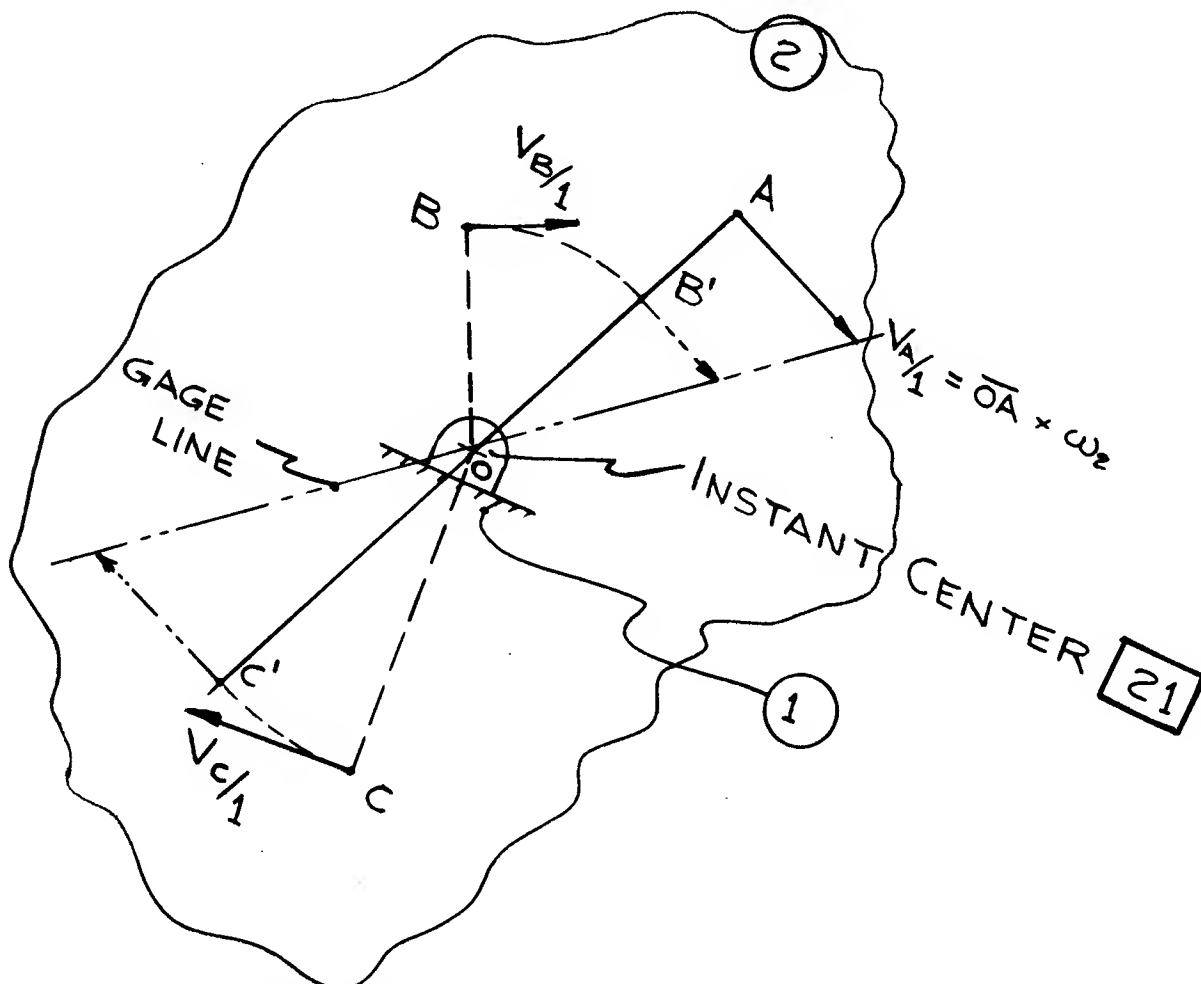
Given link (2) rotating about a fixed frame, (link (1)) we can locate the fixed instant center 21 of (2) with respect to (1) by inspection as shown. If we know the velocity of any point "A" (its direction is perpendicular to line OA and its magnitude is equal to the product of $\overline{OA}$ and the angular velocity of (2) with respect to (1)) we can determine the magnitude and direction of velocity for any other points such as B, C by constructing a "gage" line which connects instant center 21 to the tip of the vector which represents the velocity of point A with respect to the frame. Then using the point 21 as a center we rotate points B and C onto the line joining point 21 to point A. Let's call these new points B' and C'. If we then erect lines perpendicular to line $\overline{OA}$ from B' and C', the intersection of these perpendiculars with the gage line will mark the end of a velocity vector for its respective point and establish its magnitude. By retracing the paths from B' and C' to B and C we can establish the direction of the velocity for B and C. Note that the sense of velocity B is opposite to that of C because they are on "opposite sides of the instant center".

If we examine a simplified sketch of Ferguson's Paradox (Exhibit 6) and label all the instant centers of rotation we can understand its action. Let's call the fixed frame link (1); the sun shaft and its integral gear link (2); the spider (or planet arm) link (3); and the planet shaft with its cluster of integral gears link (4). If we hold the planet arm (3) so that it cannot rotate and turn shaft (2), the gear which is integral with shaft (2) will turn shaft (4) about the instant center designated as 34 which in turn will cause the gears labeled (5) and (6) to turn about instant center 21. Both gears will turn in the same direction, but the angular velocities of (5) and (6) will be different for the mechanism as shown. The angular velocities can be made equal by additional compensating gears and then the motion of (5) and (6) can be used to drive two independent links in the same direction in perfect synchronism. If, now, we hold shaft (2) still and turn the planet arm it will carry shaft (4) in a circular path around shaft (2) i.e. about instant center 21. However, at all times, the instant center of (4) with respect to (2) will be on the line joining instant center 34 to instant center 32 (by Kenedy's Theorem).

Specifically it will be at the point where the line of centers intersects the pitch circles of gears (2) and (4) and we call it 24. Now, the gear which meshes with (6) and the gear which meshes with (5) are both integral with (4) and the velocity of any point on (4) may be determined by means of the graphical construction. If we look at the location of the meshing point of (4) and (5) and compare it with the meshing point of (4) and (6) we see that they are on opposite sides of the instant center 24 . This means that the sense or direction of their velocities will be opposite -- either separating or coming together,-- depending on the direction of rotation of the planet arm. We can therefore use this circumstance to drive two independent links in opposite directions. Also, an epicycle gear train will produce the two motions just described (synchronous unidirectional and opposite) as superimposed motions if we simultaneously drive the planet arm and the sun shaft.
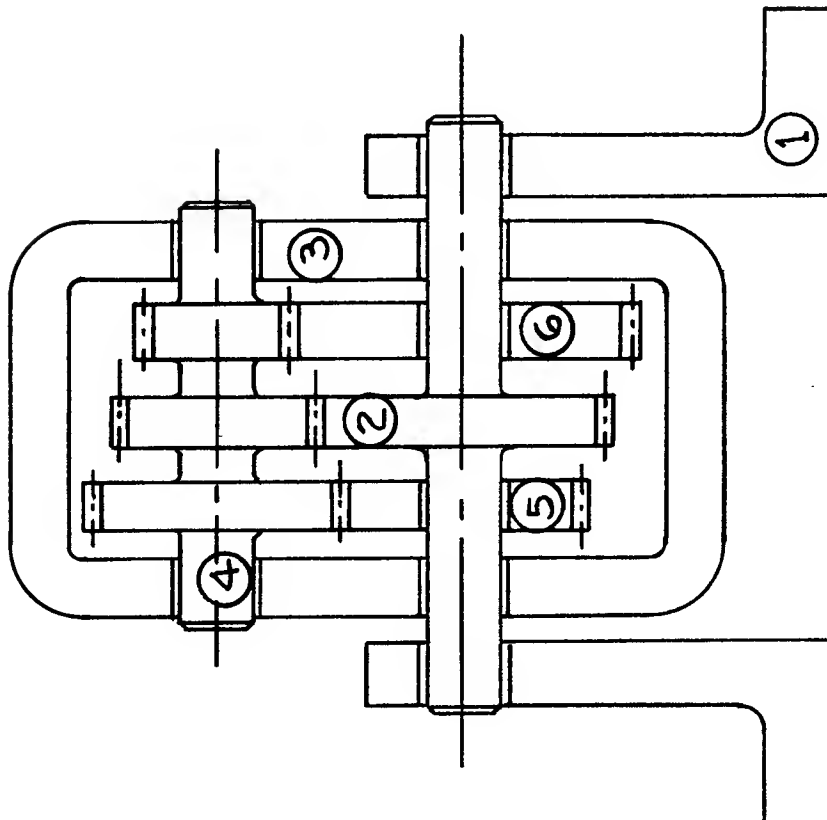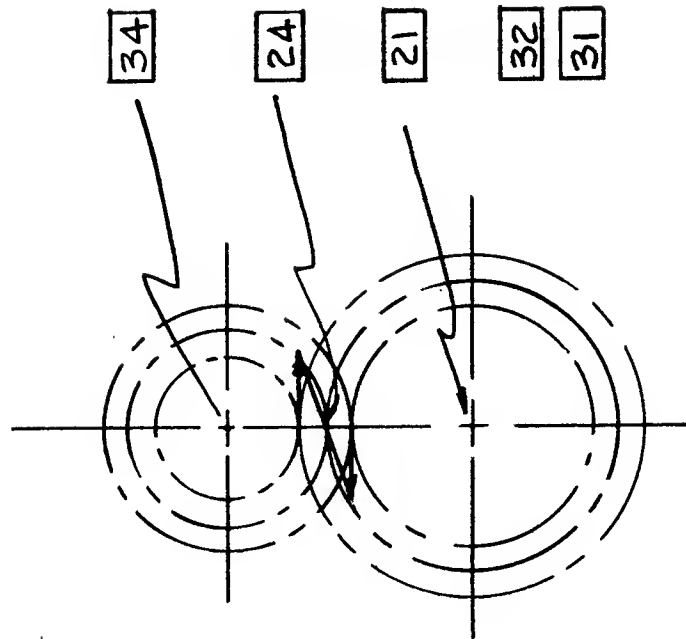
34

24

21

32

31

①

③

⑥

②

⑤

④

Exhibit 6
Simplified Sketch of Ferguson's Paradox